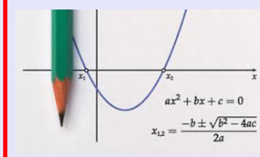
Math 125

Spring 2022

Lecture 29



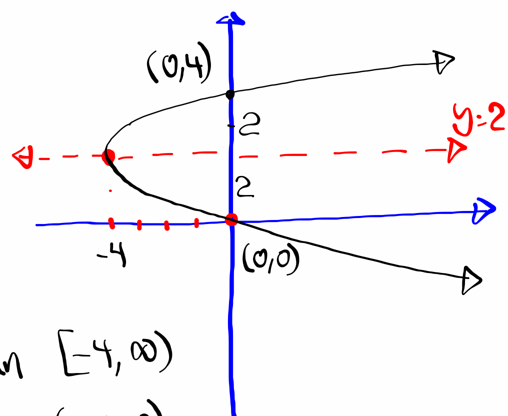
Graph $x=(y-2)^2-4$

$x=a(y-k)^2+h$

$a=1 \Rightarrow$ opens to the right

$h=-4$ Vertex $(-4,2)$

$k=2 \Rightarrow$ A.O.S. $y=2$



x-Int $(0,0)$

Y-Int $(0,0),(0,4)$

Domain $[-4,\infty)$

Range $(-\infty,\infty)$

Graph $x = y^2 + 4y + 4$

$x = ay^2 + by + c$

$a = 1 \rightarrow$ opens to the right

$b = 4$ $\quad k = \frac{-b}{2a} = \frac{-4}{2(1)} = \frac{-4}{2} = -2$

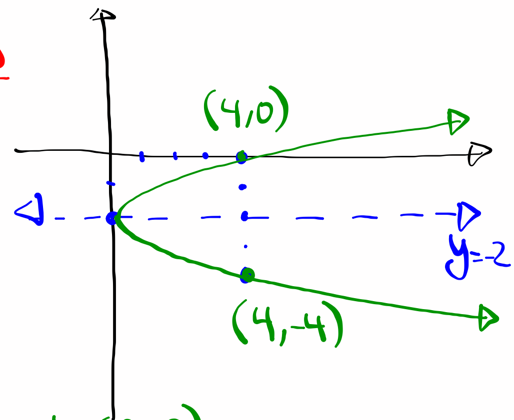
$c = 4$ $\quad h =$ plug in k

$h = (-2)^2 + 4(-2) + 4 = 0$

Vertex $(h,k) = (0,-2)$

A.O.S. $y = k$ $\quad y = -2$

x-Int $(4,0)$



Y-Int $(0,-2)$

Domain: $[0,\infty)$

Range $(-\infty,\infty)$

Graph $x = -y^2 + 2y - 6$

$x = ay^2 + by + c$

$a = -1 \rightarrow$ opens left

$b = 2$ $\quad k = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

$c = -6$ $\quad h = -(1)^2 + 2(1) - 6$

$= -1 + 2 - 6 = -5$

Vertex $(h,k) = (-5,1)$

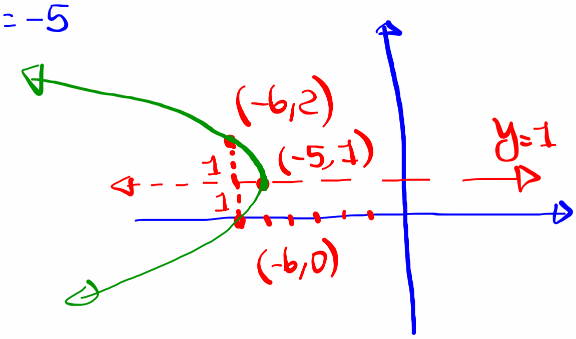
A.O.S. $y = k$ $\quad y = 1$

x-Int $(-6, 0)$

No Y-Int

Domain $(-\infty,-5]$

Range: $(-\infty,\infty)$



find

1) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$

2) $_8C_5 = \dfrac{8!}{5! \cdot (8-5)!} = \dfrac{8!}{5! \cdot 3!} = \dfrac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{5!} \cdot \cancel{3 \cdot 2} \cdot 1} = \dfrac{56}{1} = \boxed{56}$

3) $_6P_3 = \dfrac{6!}{(6-3)!} = \dfrac{6!}{3!} = \dfrac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} = 6 \cdot 5 \cdot 4 = \boxed{120}$

4) $\dbinom{9}{4} = {}_9C_4 = \dfrac{9!}{4! \cdot (9-4)!} = \dfrac{9!}{4! \cdot 5!} = \dfrac{9 \cdot \overset{2}{\cancel{8}} \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{5!}}$

$= \dfrac{9 \cdot 2 \cdot 7}{1} = \boxed{126}$

Binomial Expansion

$(a+b)^n$, $a+b \neq 0$, n is $\underbrace{\text{whole number}}_{0,1,2,3,\ldots}$

$(a+b)^n =$

$\binom{n}{0} a^n \cancel{b^0} + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n} \cancel{a^0} b^n$

$1 \qquad\qquad\qquad\qquad 1$

$(a+b)^4$

$\binom{4}{0} a^4 b^0 + \binom{4}{1} a^3 b^1 + \binom{4}{2} a^2 b^2 + \binom{4}{3} a^1 b^3 + \binom{4}{4} a^0 b^4$

1 4 6 4 1

$\binom{4}{2} = {}_4C_2 = \dfrac{4!}{2! \cdot (4-2)!} = \dfrac{4!}{2! \cdot 2!} = \dfrac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2!}}{\cancel{2!} \cdot \cancel{2} \cdot 1} = 2 \cdot 3 = \boxed{6}$

$$\boxed{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$$

There are $(n+1)$ terms

$(4+1)$ terms $\Rightarrow$ 5 terms

Degree of each term $= n$

$a^4 \to$ Deg. $= 4$

$a^3b \to$ Deg. $= 3+1 = 4$

$a^2b^2 \to$ Deg. $= 2+2 = 4$

$ab^3 \to$ Deg. $= 1+3 = 4$

$b^4 \to$ Deg. 4

Expand $(a+b)^6$

- 7 terms
- Deg. of each term = 6

$$\binom{6}{0}a^6b^0 + \binom{6}{1}a^5b^1 + \binom{6}{2}a^4b^2 + \binom{6}{3}a^3b^3 + \binom{6}{4}a^2b^4 + \binom{6}{5}a^1b^5 + \binom{6}{6}a^0b^6$$

1 6

$_6C_2 = \frac{6!}{2!\cdot 4!} = 15$

$\binom{6}{3} = {_6C_3} = \frac{6!}{3!\cdot 3!} = 20$

6 1

$$\boxed{a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6}$$

Find the $\boxed{\text{first 4 terms}}$ of $(x^3+2)^{10}$

$a = x^3$ $b=2$

$(a+b)^{10}$

$$\binom{10}{0}a^{10} + \binom{10}{1}a^9b + \binom{10}{2}a^8b^2 + \binom{10}{3}a^7b^3$$

1 10 45 $_{10}C_3 = \frac{10!}{3!\cdot 7!} = 120$

$$a^{10} + 10a^9b + 45a^8b^2 + 120a^7b^3$$

$$= (x^3)^{10} + 10\cdot(x^3)^9\cdot 2 + 45(x^3)^8\cdot 2^2 + 120(x^3)^7\cdot 2^3$$

$$= \boxed{x^{30} + 20x^{27} + 180x^{24} + 960x^{21}}$$

Find the $\boxed{\text{first 3 terms}}$ of $(2x-3)^6$

$a=2x$ $b=-3$

$(a+b)^6$

$\binom{6}{0}a^6+\binom{6}{1}a^5b+\binom{6}{2}a^4b^2$

1 6 15

$a^6+6a^5b+15a^4b^2$

$(2x)^6+6(2x)^5(-3)+15(2x)^4(-3)^2=$

$=2^6x^6+6\cdot 2^5x^5\cdot(-3)+15\cdot 2^4x^4\cdot 9=$

$=\boxed{64x^6-576x^5+2160x^4}$

How to find $(k+1)$th term of $(a+b)^n$

$$\binom{n}{k}a^{n-k}b^k$$

Find $\boxed{\text{the 4th term}}$ of $\boxed{(a+b)^9}$

$k+1=4$ $k=3$

$\binom{9}{3}a^{9-3}b^3=\boxed{84a^6b^3}$

Find the 6th term of $(a+b)^{12}$

$k+1=6\to k=5$

$\binom{12}{5}a^{12-5}b^5=\boxed{792a^7b^5}$

Find the 8th term of $(a+b)^{10}$

$k+1=8$

$k=7$

$\binom{10}{7}a^{3}b^{7} = \boxed{120\,a^{3}b^{7}}$ ($10-7$)

Find the 4th term of $(x^5+2)^8$

$a=x^5$ $b=2$

4th term of $(a+b)^8$

$k+1=4$

$k=3$

$\binom{8}{3}a^5 b^3 = 56(x^5)^5(2)^3$

$= 56 \cdot x^{25} \cdot 8 = \boxed{448x^{25}}$

Find the 6th term of $(x^4 - y^3)^{11}$

$a=x^4$ $b=-y^3$

6th term of $(a+b)^{11}$

$k+1=6$

$k=5$

$\binom{11}{5}a^6 b^5 = 462(x^4)^6(-y^3)^5$ ($11-5$)

$=462\,x^{4\cdot 6}\cdot -y^{3\cdot 5}$

$=\boxed{-462\,x^{24}y^{15}}$

Find the 5th term of $\left(\frac{1}{2}x - 4y^2\right)^{10}$

$a = \frac{1}{2}x$ $\quad$ $b = -4y^2$

5th term of $(a+b)^{10}$

$k+1 = 5$

$k = 4$

$$\binom{10}{4} a^6 b^4 = 210\left(\frac{1}{2}x\right)^6\left(-4y^4\right)^4$$

$$= 210 \cdot \frac{x^6}{2^6} \cdot (-4)^4 (y^4)^4$$

$$= \frac{210 \cdot 256}{64} x^6 y^{16}$$

$$= \boxed{840\, x^6 y^{16}}$$

$\sum$ Summation

$$\sum_{n=1}^{5} (2n+1) = \underset{n=1}{(2\cdot 1+1)} + \underset{n=2}{(2\cdot 2+1)} + \underset{n=3}{(2\cdot 3+1)} + \underset{n=4}{(2\cdot 4+1)} + \underset{n=5}{(2\cdot 5+1)}$$

$$= 3 + 5 + 7 + 9 + 11$$

$$= \boxed{35}$$

$$\sum_{i=1}^{6} [i^2 - 2i] = \underset{i=1}{[1^2 - 2\cdot 1]} + \underset{i=2}{[2^2 - 2\cdot 2]} + \underset{i=3}{[3^2 - 2\cdot 3]} + \underset{i=4}{[4^2 - 2\cdot 4]} + \underset{i=5}{[5^2 - 2\cdot 5]} + \underset{i=6}{[6^2 - 2\cdot 6]}$$

$$= -1 + 0 + 3 + 8 + 15 + 24$$

$$= \boxed{49}$$

Find $\sum_{i=1}^{5} (i^3 - i^2)$

$= (1^3 - 1^2) + (2^3 - 2^2) + (3^3 - 3^2) + (4^3 - 4^2) + (5^3 - 5^2)$

$= 0 + 4 + 18 + 48 + 100$

$= \boxed{170}$

Final Exam:

1) June 2, 2022, 7:00-9:00, Thursday

   You can arrive early (6:30 AM) and

   stay as late as (10:00 AM).

   You must arrive no later than 7:20.

2) Same Process as exams 1 & 2.

3) No emails after the final until You hear from me.

4) Review notes, class quizzes, SG, and exams.

5) No more lectures, but I hold my office hrs.